RESEARCH PROJECT: THE GEOMETRY OF ALGEBRAIC VARIETIES

This proposal is situated in the field of algebraic geometry, roughly speaking the study of solution spaces of systems of polynomial equations; these spaces are called varieties.

The present project consists of a series of closely interrelated research problems in the field of algebraic geometry, whose numerous common links often revolve around the themes of explicit techniques for the constructions of algebraic varieties and the study of their properties and invariants.

The postdoctoral researcher will work on problems in algebraic geometry, with particular attention to the explicit construction of projective varieties, and the study of their properties and invariants. The ideal candidate will be able to successfully interact with the members of the algebraic geometry group and be competent in more than one of the members interests. Depending on the skills of the best candidate, they could work in several macro-topics, ideally working in close collaborations with several members of the group. The interdisciplinarity of the research and the independence of the postdoctoral researcher will also be positively evaluated.

Several members of our group are interested in the geometry of varieties with trivial first Chern class (Ricci flat varieties) - or *zero curvature*, that are widely investigated in algebraic and differential geometry, with applications also in high-energy physics. Thanks to the celebrated Beauville-Bogomolov decomposition theorem, the building blocks of these objects are complex tori, **Calabi-Yau (CY) and hyperkähler manifolds (HK)**. The study of the second and third classes of varieties is well represented in Bologna. We are currently investigating many research directions, including the birational geometry of hyperkähler and Calabi-Yau varieties and their mildly singular counterpart (called *irreducible symplectic varieties (ISV)* in the former case), the study of symplectic and non-symplectic automorphism on HK manifolds, elliptic and lagrangian fibrations on CY and HK, constructions of explicit projective models of HK manifolds, the ongoing study of HK manifolds of O'Grady type.

Another class of varieties - a priori very different from the previous ones - consists of **Fano varieties**. These are in fact *positively curved* - in particular their Kodaira dimension is $-\infty$. We are interested in a large class of problems about the geometry of Fano varieties. Some relevant examples include the classification and construction of Fano 4-folds, and the K-stability problem for 3-folds and beyond. There is also an intriguing link between Fano varieties and HK manifolds, given by the so-called *Fano varieties of K3 type.* This is roughly a sub-class of Fano varieties with special Hodge-theoretical properties to which one is expected to associate projective examples of HK, using moduli-theoretic constructions.

Hodge theory is also one of our core interests. There is an important focus on the study of special Hodge structures, but also the $P{=}W$ conjecture, the study of Higgs bundles and their moduli spaces, Hitchin fibrations, non-abelian Hodge theory.

Moduli-type techniques are ubiquitous in all of the above topics and especially in **deformation theory**. We are currently interested in many features of this field, including for example the moduli spaces of modular or projectively hyperholomorphic vector bundles, local problems such as formality and smoothness of moduli spaces of sheaves and deformation of toric varieties. Interestingly, the study of this area is deeply interlinked with some previously mentioned topics. As an example, compactified moduli spaces of hyperholomorphic vector bundles on HK manifolds can lead to constructions of HK and ISV varieties. Also, the study of the deformation theory of toric varieties will help probe the study of the boundary of K-stable moduli spaces of Fano varieties.

Finally, a great deal of interest is directed to **classical constructions** in algebraic geometry. These include the construction of models for the Hilbert scheme of points on special varieties, hypersurface singularities, projective duality, secant and Segre-Veronese varieties, homogeneous vector bundles on homogeneous varieties. To no one's surprise, advancements in these areas will feed back into the already mentioned topics. Just to mention few examples, the classical theory of projective duality (together with its *homological updates*) is widely used in the context of Fano varieties of K3 type, the theory of hypersurface singularities is deeply intertwined with Hodge theory and the study of special Hodge structures, and so on.

To wrap all of it up, our ideal candidate should be able to master more than one of these topics, and effortlessly use the techniques from one subfield to successfully tackle problems in another one, interacting with the many geometrical facets of our group. We also value international experience and a solid research track and appreciate a candidate with a clear research vision and a solid pathway to impact in the algebraic geometry world. We will make sure to widely publicize this position in national and international mailing lists in algebraic geometry, in order to attract the best possible talents who might be interested in working with us.